Theory of Homes and Jobs [Preliminary. For Part II paper] 18 September 1997

Andrew Oswald, University of Warwick. a.j.oswald@warwick.ac.uk

The December 1996 working paper "A Conjecture on the Explanation for High Unemployment in the Industrialized Nations: Part I" provides simple evidence that unemployment, u, and home ownership, h, are correlated (with gradient du/dh = 0.2).

Since then, further evidence includes the following:

(i) Region cross-sections for Canada, Australia and Finland also generate du/dh = 0.2.

(ii) An unbalanced panel of 19 OECD countries, from 1960 to 1990, produces a coefficient on home ownership of 0.17. This emerges in fixed and random effects models. The regressions include controls for union density and unemployment benefit generosity, and year dummies.

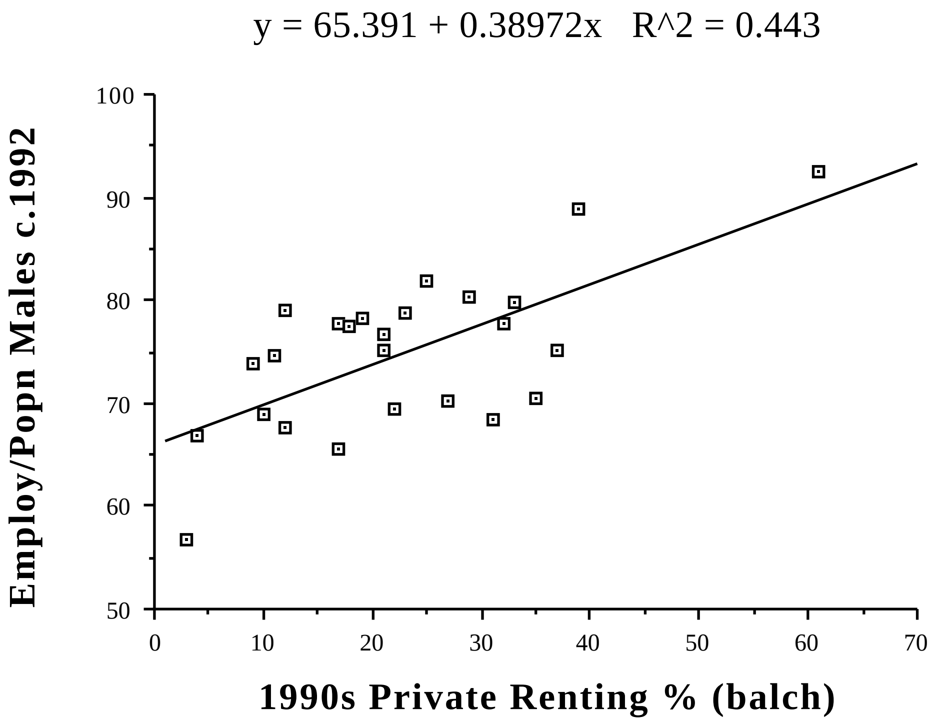
(iii) The same result is found in a random effects model using data on 20 OECD countries for the mid-1980s and early 1990s. Here, apart from home ownership, the controls include countries' benefit replacement rates, benefit duration, labour market programme coverage, union coverage, employment protection, coordinated bargaining, inflation, a period dummy, tax rate, and union density. Steve Nickell kindly provided most of these data.

(iv) Using data on Britain's regions, there is evidence that the size of the private rental sector is correlated with the number of inter-regional movers per head, the height of the UV curve, and the average outflow rate from unemployment. Using micro data for Britain, job satisfaction is highest among renters, commuting times are longest for home owners, and it is home owners who express the least willingness to move when asked what they would do if they lost their jobs.

(v) The plot overleaf provides another way to depict the cross-section relationship.

Figure X

The Relationship Between Countries' Employment Rates and the Size of their Private Rental Housing Markets: 25 Industrial Nations in the 1990s



The vertical axis is the male employment rate, expressed as a proportion of population. This is for men of ages 15-65. These data are for circa 1992 and come mostly from the OECD Employment Outlook, July 1995, Table A, 1992 column, p.204. For the Czech Republic, Hungary and Poland, the data are for 1993, and come from Table B, p.164, of OECD Employment Outlook July 1997.

The data on housing are principally from Table 1.12, page 11, of Housing Policy in Europe, edited by Paul Balch, Routledge, London and New York, 1996. For the Czech Republic, Hungary and Poland, the data come from the later pages of Balch, including p.286. For New Zealand and Australia, the data are taken from p.184 of From Public Housing to the Social Market, edited by J. Kemeny, Routledge, London and New York, 1995. Marion Steele of the University of Guelph provided helpful information about Canada. Japanese and US data were imputed by making a small adjustment to (1 - home ownership rate), calculated from UN census data. The countries are Canada (renting % = 34, Employ/pop % = 69.5), Switzerland (60, 91.4), Japan (38, 87.8), Sweden (16, 76.7), Norway (18, 77.3), France (21, 68.5), Australia (20, 75.7), USA (32, 78.8), Netherlands (17, 76.5), UK (10, 73.6), W Germany (36, 74), Austria (22, 77.8), Belgium (30, 67.3), Denmark (24, 80.7), Finland (11, 66.6), Luxembourg (31, 76.8), New Zealand (20, 74), Ireland (9, 67.8), Italy (8, 72.9), Spain (16, 64.6), Greece (26, 69.1), Czec Rep (11, 78.1), Hungary (2, 55.6), Portugal (28, 79.4), Poland (3, 65.9).

Omitting the two most favourable observations, Switzerland and Hungary, alters the gradient to 0.26.

A number of ideas motivate the analysis.

1. When someone buys a home, they invest in immobility. Because it is then costly to move, an owner takes on the risk of local area demand shocks in a more severe way than those who rent. The return to their investment comes presumably in the non-pecuniary enjoyment of owning per se, plus the ability to alter their accommodation in idiosyncratic ways. Renters cannot enjoy these pleasures. But they are freer to leave if the area is hit by a bad demand shock.

2. People choose between renting and owning. In equilibrium the two must offer the same expected utility.

3. Home owners can, if necessary, resort to a form of quasi-mobility. They can commute to a job elsewhere.

4. Commuters get in each other's way. Having a large number of commuters generates congestion costs.

5. Politicians appear to believe that home ownership should be subsidised. It would be useful to have a way of deciding whether a free market will tend to produce too little (or too much) home ownership to be socially optimal.

1. A Framework

Consider two cities, City West and City East. The cities are joined by a road. Because of the terrain, people have to live in one city or the other. They cannot live between.

The economy experiences real shocks to demand. There is ex ante uncertainty about these -- both about their location and intensity. Productivity, as measured by the real selling price of output, is denoted p. It is distributed according to a density function, g(p). Assume that prices lie in the range [0, p_2]. For simplicity, imagine that one city will obtain a draw from this distribution, and that, with certainty, the other will obtain a selling price of zero. One area will be prosperous; the other will be economically depressed. It is not known in advance, however, which city will obtain the draw from the g(p) distribution.

Before the demand shock is revealed, people must allocate themselves across space. It is this that gives the problem its character. They have to choose one city or the other. Initial location, in other words, cannot be state-contingent.

Individuals may choose to rent or buy. Landlords earn zero supernormal profit, and there are no imperfections in capital markets, so the financial return from renting and owning is the same. Home ownership, however, gives a direct additional supplement to utility. Denote it i, 'pride of ownership'. This might be viewed as the utility from a sense of ownership per se, or as capturing the freedom of owners to make idiosyncratic alterations to their home in a way that rental tenants cannot, or as somehow measuring a sense of security engendered by ownership. Where y is income, the utility of a home owner is denoted $u(y^h + i)$. By contrast, the utility of a renter is $u(y^r)$.

Those who choose to rent, rather than buy, gain in a different way. They invest in ex post flexibility. Renters, by assumption, can move city costlessly if and when they wish. Hence, if a bad shock strikes their city, such individuals move to the prosperous area. Home owners have to pay a moving cost, k^m, if they wish to switch locations. They have one alternative. Home owners who have bought in what transpires to be the unlucky location may commute daily to a job in the prosperous city. This incurs cost k(c), where k(.) is a convex increasing function, and c is the total number of commuters using the road.

Let r be the number of renters. Let h be the number of home owners. By choice of units, fix r + h =1. For simplicity, let the two cities have the same amenities, so that workers have no intrinsic preference between the two.

If the government is optimizing, it can choose its other policies to be state-contingent. But it cannot fix functions r(p) and h(p). They are scalars chosen before the realisation of price, p.

Population is unity. Employment is n. There is a concave production function f(n), so real output is pf(n). The value of leisure or self-employment income is b. The number of movers (those changing cities) is m.

In this world, people will spread themselves equally between the cities before the demand shock is drawn. After the draw, the real selling price of output, p, is known, and its location is known. Those home owners living in the prosperous city -- the one with a positive p -- are now in a fortunate position. They can find work costlessly without moving or commuting, and they also enjoy pride of home ownership. Renters in the depressed city are able to move without cost, so they do so, and find work in the prosperous city. Home owners in the depressed city, however, have to incur either cost k(c) as a commuter, or k^m as a mover. By assumption, wages adjust in a competitive manner.

In this world, the reservation wage schedule of workers is kinked at two points. It initially runs horizontal at level b, the value of leisure. Once employment reaches a point equal to the sum of all renters plus half the home owners, there are no more workers available at reservation wage b. Then the reservation wage schedule rises, tracing out the function b+k(c). Over this zone, home owning commuters from the depressed city are attracted in at a wage just large enough to compensate for the additional cost of commuting. As employment grows, gradually the roads become full of commuters, and the cost of commuting between cities, k(c), increases until it reaches k^m, which is the cost of migrating from the depressed city to the prosperous one. A second kink is then reached. The reservation wage schedule turns horizontal again, at $w = b+k^m$. Firms may once more obtain labour perfectly elastically.

It is natural to denote these three zones as *slump*, *boom*, and *strong boom.* In the first, the reservation wage is flat at b, and the economy needs no labour mobility. In the second zone, the marginal product of labour is fairly large, commuting occurs, and the reservation wage rises as employment increases. In the third zone, the reservation wage is flat at the value of leisure plus moving cost. The value of the marginal product of labour is so high that migration is required to attain equilibrium.

Partly because real governments intervene heavily in housing markets, it is useful to begin by exploring the social planning problem.

Consider the government's task in such a world. It must attempt to maximize social wellbeing, which is taken here to be described by the sum of individual utilities. To do so, the government can be thought of as setting two sets of variables. First, in a non-contingent way, it fixes the numbers of home owners, h, and renters, r. Second, in a contingent way, it implicitly fixes the number of commuters, c, the level of employment, n, the incomes of renters and home owners, y^r and y^h, and the number of those moving from the depressed city to the boom city, m.

The government's objective is to maximize expected social welfare

$$W = \int_{0}^{p2} \left\{ru(y^{r}) + hu(y^{h} + i)\right\} g(p)dp \qquad (1)$$

by choice of h, r, c(p), n(p),y^r(p), y^h(p), m(p). It must do so subject to the following constraints:

$1 - n \geq 0$ (2)

$c \geq 0$ (3)

$m \geq 0$ (4)

$$\int_{0}^{p2} \left\{ry^{r} + hy^{h}\right\} g(p)dp \leq$$
$$\int_{0}^{p2} \left\{pf(n) + (1 - n)b - k(c)c - k^{m}m\right\} g(p)dp \qquad (5)$$

$m + c \geq n - r - h/2$ (6)

$1 = h + r$ (7).

Equations (2)-(4) state that employment, n, must be less than or equal to unity (the population), and the numbers of commuters, c, and inter-city migrants, m, cannot be negative. Equation (5) is the real resource constraint: expected real income is weakly less than expected real output. The right hand side of (5) includes leisure valued in income units. Equation (6) defines who is available for work. It can be thought of as saying that employment must be less than or equal to the sum of the movers, plus the commuters, plus all the renters, plus half the home owners (h/2). The last term is because half the home owners in the economy automatically live in the prosperous region. In a slump, n may lie below r+h/2; then (6) holds as an inequality. Finally, (7) is a population constraint.

Attach the following multipliers to these constraints:

$\psi, \alpha, \beta, \varphi, \mu, \gamma$

and write the first-order conditions as

h: $Eu(y^h + i) - \varphi Ey^h + \frac{\mu}{2} - \gamma = 0$ (8)

n(p): $-\psi + \varphi[pf'(n) - b]g(p) - \mu = 0$ (9)

c(p): $\alpha - \varphi[k'(c) + k(c)]g(p) + \mu = 0$ (10)

r: $Eu(y^r) - \varphi Ey^r + \mu - \gamma = 0$ (11)

$y^r(p)$: $u'(y^r) - \varphi = 0$ (12)

$y^h(p)$: $u'(y^h + i) - \varphi = 0$ (13)

m(p): $\beta - \varphi k^m g(p) + \mu = 0$ (14)

These lead to a number of ideas.

<u>Proposition 1</u> <u>For any demand shock, the greater is the pride of home ownership, i, the lower is the equilibrium level of employment in the economy</u>.

By first order conditions (12) - (13), there is so-called full insurance, so that renters and home owners have the same marginal utility of income in every state of nature. Trivially, the expected utility of renters then equals that of home owners. Hence, subtracting equation (8) from equation (11),

$\varphi E(y^h - y^r) + \frac{\mu}{2} = 0.$ (15)

By the first order conditions, y^h and y^r are constant over different states of nature and $u(y^h$ and $i) = u(y^r)$. Thus

$y^h - y^r = -i.$ (16)

Therefore equation (15) reduces to

$-\varphi i + \frac{\mu}{2} = 0.$ (17)

Substituting this into equation (9), eliminating the terms in the multiplier μ, gives at an interior optimum

$$-2\phi i + \phi[pf'(n) - b]g(p) = 0. \qquad (18)$$

Rewriting this, there is a wedge between the value of the marginal product of labour and the value of leisure, given by:

$$pf'(n) - b = \frac{2i}{g(p)}. \qquad (19)$$

Equation (19) reveals one simple feature of the model. Pride of home ownership, i, acts here like the value of leisure (or unemployment benefit). It supplements b. Not surprisingly, then, the effect of home ownership in the economy is to drive up reservation wages. The integer 2 should not be viewed as general; it enters the formula by the assumption of two cities.

Consider, in turn, what happens in a strong boom and an ordinary boom.

When there is a *strong boom*, selling prices are high enough to induce firms to pay some home owners to move. Consider an interior optimum of this sort, where commuters c>0 and movers m>0. Then, adding together first order conditions (9) and (10), noting that inequalities (2)-(4) will in general not bind, and using the fact that commuting will not exceed the point where labour can be obtained more efficiently by having migration, gives:

$$pf'(n) - b = k'(c)c + k(c) = k^m. \qquad (20)$$

Here the value of the marginal product of labour is set equal to [b+k'(c)c + k(c)], which is the value of leisure plus the cost externality generated by commuters plus the direct cost of commuting. The value of the marginal product of labour is also equal to [b + k^m], which is the value of leisure plus an individual's cost of moving to the prosperous city.

The corollary to this is that, in a strong boom, the greater are mobility costs, k^m, the lower is employment. This simply follows from the concavity of the production function. When it is expensive to move workers who have bought a house in the wrong region, the economy's efficient employment level is lower.

Consider now a *boom* equilibrium, where there is commuting but no moving. It is easy to see that equilibrium employment will be a decreasing function of the amount of home ownership, h, in the economy. By the first order conditions used earlier

$$pf'(n) - b = k'(c)c + k(c). \quad (21)$$

Writing this out in full, remembering m = 0 in this case, gives

$$pf'(n) - b = k'(n - 1 + \frac{h}{2})(n - 1 + \frac{h}{2}) + k(n - 1 + \frac{h}{2}). \quad (22)$$

Differentiating implicitly:

$$\frac{\partial n}{\partial h} = \frac{1}{2}\frac{ck''(c) + 2k'(c)}{pf''(n) - ck''(c) - 2k'(c)} < 0. \quad (23)$$

Equation (23) summarises the trade-off between homes and jobs. This trade-off holds even in an economy where the government is behaving optimally (intuitively, because governments cannot banish real mobility costs). Home ownership lowers employment by increasing the reservation wage of individuals who turn out -- after the spatial demand pattern is known -- to be in the wrong place at the wrong time. The size of the drop in employment as home ownership grows is determined by the convexity of k(.), the concavity of f(.), and the nature of the density function, g(.).

Equation (23) has another implication. A one percentage point rise in home ownership is associated with less than a half a percentage point fall in employment. This is ultimately because there is an extra term, pf "(n), in the denominator of (23).

A later section of the paper considers equilibria where there is no government intervention. In passing, however, it can be seen that socially efficient employment will typically differ from the free market level. Privately people will ignore the congestion effects they create on other commuters, setting

$$pf'(n) - b = k(c), \quad (24)$$

while instead the socially efficient level allows for the extra commuting externality term, $k'(c)c$.

A natural question to ask is whether there is a case for governments to subsidise those who buy their own homes. Again, this question is considered more fully later, but one argument is clear from this analysis. Because home ownership conveys psychic utility, it is likely to be optimal to tax it. This is immediate from

$$u'(y^h + i) = u'(y^r) \quad (25)$$

and the positivity of ownership-pride i.

A central question is whether the employment-depressing effect of high home-ownership levels is true in some overall sense. Expected total employment in the economy seems a natural criterion.

<u>Proposition 2</u> <u>Expected employment in the economy is a decreasing function of the amount of home ownership</u>.

To see why, write expected employment as

$$En = \int_0^{p0} n(b, p)g(p)dp + \int_{p0}^{p1} n(b, h, k^m, p)g(p)dp + \int_{p1}^{p2} n(b, k^m, p)(g(p)dp . \quad (26)$$

Each integral corresponds to each of the three zones (slump, boom, strong boom) discussed earlier. Three different employment functions are used -- again corresponding to the first order conditions described earlier. The price levels p_0 and p_1 are functions of home ownership, h. Differentiating throughout, noticing that h enters only the second of the three integrals,

$$\frac{\partial En}{\partial h} = \int_{p_0}^{p_1} n_h(b, h, k^m, p)g(p)dp \ + \text{end point terms} \quad (27)$$

$$= \int_{p_0}^{p_1} \frac{1}{2} \frac{ck''(c) + 2k'(c)}{pf''(n) - ck''(c) - 2k'(c)} g(p)dp < 0. \quad (28)$$

The end-point terms, which are the derivatives of the upper and lower price levels in the integrals, sum to zero (the proof is omitted, but a later section gives details in a slightly different setting). Expression (28) is unambigously negative. It is the analogue of (23) earlier. Intuitively, all other integral terms cancel because the home ownership level does not affect the heights of the two horizontal portions of the reservation wage schedule. In the slump and strong boom zones, in other words, a change in home ownership has locally no effect.

2. The Housing and Labour Markets Without Optimal Intervention

Western governments may not approach the optimal form of intervention described above. They do, however, offer subsidies to home ownership. These governments appear to believe that the free market would produce too few owners.

This section begins by calculating the effect on home ownership of a subsidy, s, for home owners. Utility of a home owner is now $u(y^h + i + s)$.

The analysis uses several steps.

Step 1 *In zone p_0 to p_1, there is an inverse correlation, for a given demand shock, between employment, n, and home ownership, h.*

This is a special case of the algebra used earlier for optimal intervention. In this boom zone, the marginal product of labour equals the sum of the value of leisure b and commuting cost c. Hence

$pf'(n) = b + k(c)$ (29)

or in full

$pf'(n) = b + k(n - 1 + h/2)$ (30)

because the number of commuters is determined by

$n = r + c + m + h/2$ (31)

and

$1 = h + r.$ (32)

Thus

$n = 1 - h + c + m + h/2$ (33)

whence

$c = n - 1 + h/2$ (34).

Differentiating through (30),

$$\frac{\partial n}{\partial h} = \frac{1}{2} \frac{k'(c)}{pf''(n) - k'(c)} < 0. \quad (35)$$

As before, though now without the externality term, there is a local trade-off between jobs and homes.

<u>Step 2</u> *In both slump zone 0 to p_0 and strong-boom zone p_1 to p_2, there is no correlation, given a price shock p, between employment and home ownership.*

This is as before. In the zone between price zero and price p_0,

$pf'(n) = b.$ (36)

In zone p_1 to p_2,

$pf'(n) = b + k^m.$ (37)

Step 3 *The greater is home ownership, ceteris paribus, the greater is a renter's expected utility.*

Renters are assured of a job, so face little uncertainty, except about the wage. A renter's expected utility can be written in full as

$$EU^r = \int_0^{p0} u(b)g(p)dp + \int_{p0}^{p1} u(pf\,'(n))g(p)dp + \int_{p1}^{p2} u(b + k^m)g(p)dp \qquad (38)$$

where it should be borne in mind that p_0 and p_1 are functions. Differentiating partially:

$$\frac{\partial EU^r}{\partial h} = \frac{dp_0}{dh}[u(b) - u(p^0 f\,'(n))]g(p_0) + \frac{dp_1}{dh}[u(p^1 f\,'(n)) - u(b + k^m)]g(p_1) + \int_{p0}^{p1} u'(pf\,'(n))pf\,''(n)\frac{\partial n}{\partial h}g(p)dp$$

$$= \int_{p0}^{p1} u'(pf\,'(n))pf\,''(n)\frac{\partial n}{\partial h}g(p)dp > 0, \qquad (39)$$

using the fact that the square bracket terms are zero, and also Step 1.

Step 4 *The greater is home ownership, h, the greater is a home owner's expected utility. But the rise is smaller than that in the expected utility of renters.*

A home owner's expected utility is

$$EU^h = \int_0^{p0} u(b + i + s))g(p)dp + \frac{1}{2}\int_{p0}^{p1} u(pf\,'(n) + i + s)g(p)dp$$

$$+ \frac{1}{2}\int_{p0}^{p1} u(b + i + s)g(p)dp + \frac{1}{2}\int_{p1}^{p2} u(b + k^m + i + s)g(p)dp$$

$$+ \frac{1}{2}\int_{p1}^{p2} u(b + i + s)g(p)dp \qquad (40)$$

This is a long expression -- because there are many things that can happen to an individual who owns. The first integral is the outcome in a slump: the person gets only leisure b but enjoys pride of ownership and the government's income subisdy to home buyers. The next two integrals, which occur with weights of one half, are the utilities in a boom of someone located respectively in the prosperous and depressed cities. The last two integrals in (40) are the equivalent for a strong boom. The penultimate integral in (40) describes the most favourable outcome for a home owner: this is someone who chose the right city in which to settle, receives the high-wage needed to attract migrants (namely, $b+k^m$), but also has the benefit of the pride of ownership, i, and the income subsidy, s).

Differentiating (40) partially with respect to h, eventually produces, after cancellation of a set of dp/dh terms, the following:

$$\frac{\partial EU^h}{\partial h} = \frac{1}{2}\int_{p0}^{p1} u'(pf\,'(n) + i + s))f\,''(n)n_h g(p)dp. \; > 0 \quad (41)$$

This can be compared with the expression in the last line of Step 3. By concavity of the utility function:

$$\int_{p0}^{p1} \{\frac{1}{2}u'(pf\,'(n) + i + s) - u'(pf\,'(n))\}pf\,''(n)n_h g(p)dp$$

$$< 0 \qquad (42)$$

which establishes the result.

<u>Step 5</u> *Ceteris paribus, the greater is income subsidy, s, the higher is a home owner's expected utility.*

This is immediate from the expression for a home owner's expected utility.

It is now possible to establish an unsurprising result that takes more proving than might have been anticipated.

<u>Proposition 3</u> <u>A rise in subsidy, s, leads to higher home ownership, h. This in turn depresses the expected level of employment.</u>

In a free market equilibrium, the expected utility of renters must equal that of home owners. Differentiating implicitly in such an equality:

$$\frac{\partial EU^r}{\partial h}.dh + \frac{\partial EU^r}{\partial s}.ds = \frac{\partial EU^h}{\partial h}.dh + \frac{\partial EU^h}{\partial s}.ds \qquad (43)$$

which implies, using the steps above, and the fact the derivative of a renter's expected utility with respect to the income subsidy is zero, that

$$\frac{\partial h}{\partial s} = \frac{\frac{\partial EU^h}{\partial s}}{\frac{\partial EU^h}{\partial h} - \frac{\partial EU^r}{\partial h}} > 0. \qquad (44)$$

The inequality follows from the various parts proved in the steps above.

Intuitively, equilibrium occurs at the intersection of two upward-sloping functions in expected utility/home ownership space. One, EU^r, defines the expected utility of renters; the other, EU^h, that of owners. The former is steeper than the latter. Thus a shift up in the EU^h schedule increases the amount of home ownership in equilibrium.

The employment-depressing effect of home ownership is then simply a variant on the process described earlier. It is straightforward to check that here

$$\frac{\partial En}{\partial h} = \int_{p0}^{p1} \frac{1}{2} \frac{2k\,'(c)}{pf\,''(n) - 2k\,'(c)}\, g(p)dp < 0. \quad (45)$$

Finally, in the previous section, both home owners and renters whether in work or jobless all enjoyed the same utility ex post, because the government set taxes and subsidies to ensure that. This is no longer true, of course, in the present case of limited intervention.

Proposition 4 (i) Except in the slump zone, the level of utility of those in work is higher, on average, than those without jobs. (ii) Job satisfaction is higher among renters. Satisfaction with accommodation is higher among home owners.

Part (i) is immediate from the fact that, by not having to pay commuting or moving costs, some employees earn supernormal returns, that is, $u > u(b)$. Part (ii) follows from the fact that renters earn supernormal returns while home owners receive the supplement i from pride of ownership. In a sense, renters get more of their utility from work, while home owners get more from where they live.

3. Optimal Home Ownership

It is straightforward to show the following.

Proposition 5 In a free market, home ownership differs from its socially optimal level.

There are two forces at work. One tends to make home ownership too high; the other tends to make it too low.

First, congestion externalities act to make home ownership higher than socially optimal. Those commuting to work after a bad demand shock in their city ignore the externalities they create for other commuters. Second, assuming a missing insurance market in regional demand shocks, individuals in a world without government intervention will become renters too readily. This is a rational strategy individually, because, by renting rather than buying, a person invests in the ability to respond flexibly to a low regional demand for labour.

To illustrate the argument, consider a simple case where g(p) is degenerate at a real price, p, at which there will be some commuting. Compress the model, using the earlier notation, into the problem:

Maximize $W = hu(y^h + i) + (1 - h)u(y^r)$

subject to $pf(n) + (1 - n)b - k(n - 1 + h/2)(n - 1 + h/2) \geq hy^h + (1 - h)y^r$. It follows that at a social optimum:

$$k'(.)(n - 1 + h/2) + k(.) = 2i \qquad (46)$$

and

$$pf'(n) = b + 2i. \qquad (47)$$

Equation (47) is a special case of the earlier equation (19).

In a free market, however, the equality of expected utilities of renters and home owners ensures:

$$u(pf'(n)) = \frac{1}{2}u(pf'(n) + i) + \frac{1}{2}u(pf'(n) - k + i) \qquad (48)$$

where the left hand side is a renter's guaranteed utility and the right hand side is a home owner's utility averaged across the lucky and unlucky cities. Rewriting (48),

$$u(pf'(n) + i) - u(pf'(n)) = u(pf'(n)) - u(b + i). \qquad (49)$$

Therefore, if u(.) is concave,

[pf '(n) + i] - pf '(n) ≥ pf '(n) - [b + i]   (50)

or simply

pf '(n) ≤ b + 2i.   (51)

Hence employment, n, exceeds that at a full social optimum, n*. This means that in the free market outcome the commuting cost function k = k(n - 1 + h/2) lies -- in cost/home ownership space -- everywhere above its equivalent socially optimal k(n* - 1 + h/2) function. But in equilibrium pf '(n) = b + k. This plus (51) implies k < 2i in the free market outcome.

Consider one extreme example: that with congestion costs and a full set of insurance markets (workers are then effectively risk-neutral). Thus the utility function is linear and (50) holds as an equality and not a strict inequality. Employment is then automatically at its socially optimal level of pf '(n) = 2i + b.

The other optimal rule, derived earlier, is k = 2i. As the free-market k(.) function lies below the socially optimal k'(.)c + k function, home ownership, h, increases to the point where it exceeds the desirable level, h*. Too many home owners exist, and, through their commuting, create too much congestion.

Think of the other extreme. If congestion costs are zero, so that k'(c) = 0, and there is a missing insurance market in regional shocks, free-market home ownership might be too low. Here utility is concave and equation (46) reduces to k = 2i at the social optimum. The free market does not now produce the socially optimal employment level. The inequality in (51) applies instead: employment is too high and too few people are enjoying the pride of ownership, i.

Now in k/h space the free market k(n - 1 + h/2) function lies above the social optimum k(n* - 1 + h/2) function; the latter lies lower because it has less employment.

Moreover, $k < 2i$ in the free-market equilibrium. Combining these, free market homeownership, h, is below the socially efficient level, h*.

It is not impossible that there might be implications for policy. Some advanced countries may have high commuting congestion and fairly good insurance (including social insurance) against regional shocks. If so, according to this kind of analysis, they might qualify as cases where home ownership tends to exceed the desirable level. Some developing countries, by contrast, may have little commuting congestion alongside missing insurance markets. The result might be too little home ownership.

Appendix The kinks in the reservation wage schedule

As explained in the text, prices p_0 and p_1 are functions of the level of home ownership, h. In the case of p_0, the first zone (the slump zone) ends at n = r + h/2. This is the employment level at which firms in the prosperous city have exhausted all the immediately available labour, namely, the economy's supply of renters, r, plus the local home owners, h/2.

As the sum of renters and home owners is unity, n = r + h/2 = 1 - h/2. Moreover, the slump zone has the characteristic that the value of labour's marginal product is simply equal to the value of leisure, b. Hence p_0 must solve the equation pf '(1 - h/2) = b.

Thus a rise in home ownership reduces the critical price level:

$$\frac{dp^0}{dh} = \frac{1}{2}\frac{p^0}{f\,'(n^0)}\,f\,''(n^0) < 0. \qquad (a.1)$$

At the upper price kink, p^1, there is an equivalent expression. It is the price that solves $n(b, p, h) = n(b, k^m, p)$. In other words, it must be true both that pf '(n) = b + k(n - 1 + h/2) and pf '(n) = b + k^m. Employment at this point, denoted n^1, is therefore defined by

$$k^m = k(n - 1 + h/2). \qquad (a.2)$$

Hence

$$\frac{dn^1}{dh} = -\frac{1}{2}. \qquad (a.3)$$

But $p^1 f\,'(n^1) = b + k^m$, so

$$dp^1.f\,'(n^1) + p^1.f\,''(n^1).dn^1 = 0. \qquad (a.4)$$

When combined with equation (a.3),

$$\frac{dp^1}{dh} = \frac{1}{2}\frac{p^1 f\,''(n^1)}{f\,'(n^1)} < 0. \qquad (a.5)$$